Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Course: MPZ 3231-Engineering Mathematics IA **Assignment No.01**
Academic Year – 2013/2014

Instructions

- Answer all questions
- Write your address back of your answer scripts
- Use both sides of paper when you are doing assignment**.**
- Please send the answer scripts of your assignment **on or before the due date** to the following address.

Course Coordinator – MPZ 3231

Dept. of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka.

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You can collect model answers from virtual class ***(www.ou.ac.lk)***

User Name: student0 **Password :** MPZ3231

(1). (a). Define the scalar Triple product of three vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$.

(i) What is the geometrical interpretation of the scalar triple product of the vectors $\underline{a}, \underline{b}$ and $\underline{c}$?

(ii). Find the volume of the parallelepiped whose coterminous edges are represented by $\underline{a} = 2\underline{i} - 3\underline{j} + 4k$, $\underline{b} = \underline{i} + 2\underline{j} - \underline{k}$ and

$\underline{c} = 3\underline{i} - \underline{j} + 2\underline{k}$.

(iii). Show that the vectors $\underline{a} - 2\underline{b} + 3\underline{c}$, $-2\underline{a} + 3\underline{b} - 4\underline{c}$ and

$\underline{a} - 3\underline{b} + 5\underline{c}$ are coplanar.

Where $\underline{a}$, $\underline{b}$ and $\underline{c}$ are three non-coplanar vectors.

(b). Let $\underline{a}$, $\underline{b}$ and $\underline{c}$ be any non zero vectors

(i). Prove that $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} . \underline{c})\ \underline{b} - (\underline{a} . \underline{b})\ \underline{c}$

(ii). Using the above result, find the solution of following;

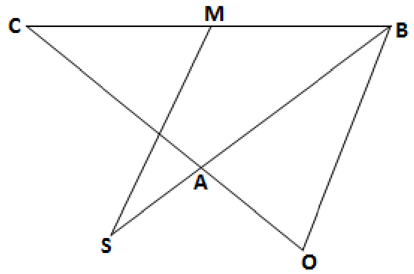
[A] $\underline{i} \times (\underline{a} . \underline{i}) + \underline{j} \times (\underline{a} . \underline{j}) + \underline{k} \times (\underline{a} . \underline{k})$

[B] $\underline{a} \times (\underline{b} \times \underline{c}) + \underline{b} \times (\underline{c} \times \underline{a}) + \underline{c} \times (\underline{a} \times \underline{b})$

(2). (a). Consider the following figure, where $\overline{OA} = \underline{a}$, $\overline{OC} = 3\underline{a}$, $\overline{OB} = \underline{b}$. M is the mid-point of BC.

(i). Express $\overline{OM}$ and $\overline{CB}$ in terms of $\underline{a}$ and $\underline{b}$.

(ii). Express $\overrightarrow{MB}$ in terms of $\underline{a}$ and $\underline{b}$.



(iii). If S lies on BA produced such that $\overline{BS} = k\,\overline{BA}$,

express $\overline{MS}$ in terms of $\underline{a}$, $\underline{b}$ and k.

(iv). Find the value of k if $\overline{MS}$ is parallel to $\overline{BO}$.

(b). Let OPQ be a triangle and let X and Y be the mid-points of OP and OQ respectively. Z is the point of intersection of XQ and YP.

If $\overline{OP} = \boldsymbol{p}$ and $\overline{OQ} = \boldsymbol{q}$, show that the position vector of Z with respect to the origin O is $\frac{1}{3}\ \boldsymbol{p} + \boldsymbol{q}$.

(3). (a). Define the vector products $\underline{a} \times \underline{b}$ of two non-zero vectors $\underline{a}$ and $\underline{b}$.

(b). Evaluate the following :

(i) $$\frac{|\underline{a} \times \underline{b}|^2 + |\underline{a} . \underline{b}|^2}{|\underline{a}|^2\ |\underline{b}|^2}$$

(ii) $\underline{a} . [\underline{b} \times (\underline{a} - \underline{b})]$

(c). $Oxyz$ is a system of right handed rectangular Cartesian co-ordinates and $\underline{i}$, $\underline{j}$, $\underline{k}$ are unti vectors along the positive directions of Ox, Oy, Oz axis, respectively.

The position vectors of the points P, Q, R with respect to the origin O are

$\underline{p} = 2\underline{j} - 3\underline{j} + \underline{k}$, $\underline{q} = \underline{i} - \underline{j} + 2\underline{k}$ and $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ respectively.

If $\overline{PQ} \times \overline{PR} = \overline{OR}$, find $|\underline{r}|$ and the position vectors $\underline{s}$ of the point S such that $PQRS$ is a parallelogram.

Deduce the area of the parallelogram $PQRS$.

(4). (a). For what real values of λ is the matix A invertible?

$$A = \begin{bmatrix} 1-\lambda & 1 & -1 \\ 2 & 1-\lambda & 2 \\ 2 & -1 & 4-\lambda \end{bmatrix}$$

(b) Show that the only real value of k, for which the following equation have non-zero solutions is 6.

$x + 2y + 3z = kx$; $3x + y + 2z = ky$; $2x + 3y + z = kz$

(c) Consider the system of linear equations

$x + y = 2$; $x + (2-\lambda)y + z = 2$; $2\lambda y + \cos\lambda\, z = 3$

Where λ is a parameter. Prove that system is inconsistent when

$\lambda(\cos\lambda + 2) = \cos\lambda$. Find the solution when $\lambda = 0$.

(5). (a). Given that $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix}$ prove that $A^3 - 4A - I = 0$ where I is the unit matrix and 0 is the null matrix of order 3.

(b). Show that the values of λ which satisfy the equation $[A - \lambda 1] = 0$ are 1, 3 and -2 where

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{pmatrix} \; and \;\; I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c). For which rational numbers a and b does the following system have

(i). No solution? (ii). A unique solution? (iii). Infinitely many solution?

$$x - 2y + 3z = 4$$

$$2x - 3y + az = 5$$

$$3x - 4y + 5z = b$$

(6). (a) If $\Delta = \begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & cd & cu(c+1) \\ 1 & ab & ab(a+b) \end{vmatrix}$ using row or column operations,

show that $\Delta = abc \begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

(b) Find the values of p for which the following system of linear equations has non-trivial solution

$$3x + y + z = px$$

$$x + 3y + z = py$$

$$x + y + 3z = pz$$

(c). Show that there is only one value for k, for which the following system of equations has a non trivial solution.

$$2x + y - z = 0$$

$$(k-2)x + ky - 2z = 0$$

$$6x + 3y + (k-1)z = 0$$

Course: MPZ 3231-Engineering Mathematics IA **Assignment No.02**
Academic Year – 2013/2014

(1). Solve the following differential equations

(a). $\cos y + (1 + e^{-x}) \sin y \frac{dy}{dx} = 0$ given that $y = \frac{\pi}{4}$ when $x = 0$

(b). $x^2(y+1) + y^2(x-1)\frac{dy}{dx} = 0$

(c). $(xy + y^2) + (x^2 - xy)\frac{dy}{dx} = 0$

(d). $(x^3 + 3xy^2)\frac{dy}{dx} = y^3 + 3x^2y$

(e). $(x - y - 1) + (4y + x - 1)\frac{dy}{dx} = 0$

(2). Solve the following differential equations

(a). $(1 + x^2)\frac{dy}{dx} + 3xy = 5x$, given that $y = 2$, when $x = 1$

(b). $\frac{dy}{dx} + y \cot x = 5e^{\cos x}$, given that $y = -4$, when $x = \frac{\pi}{2}$

(3) (a) For a horizontal cantilever of length l, with load ω per unit length,

the equation of bending is

$\mathrm{EI}\frac{d^2y}{dx^2} = \frac{\omega}{2}(l - x)^2$ where E, I, ω and l are constants.

If $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$, find y in terms of x.

Hence find the value of y when $x = l$.

(b) The equation of motion of a body performing damped forced vibrations is

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Solve this equation, given that $x = 0.1$ and $\frac{dx}{dt} = 0$ when $t = 0$.

Write the Steady – State solution in the form $K\sin(t + a)$

(4). (a). Starting from $y_1 = Ey_0$ construct the Newton's forward difference formula.

(b). Considering the function $y(x) = \sqrt{x}$, find the missing values in the following table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.00	……...						
		……...					
1.05	……...		-0.00059				
		0.02411		……...			
1.10	……...		……...		-0.00002		
		……...		……...		0.00003	
1.15	……...		……...		……...		-0.00006
		……...		……...		-0.00003	
1.20	……...		……...		……...		
		……...		0.00002			
1.25	……...		……...				
		……...					
1.30	1.14017						

(c). Find the value of $\sqrt{1.28}$ using suitable difference formula

(5). (a). Construct the Lagrange interpolation polynomial $f(x)$ of fifth degree for the data given below. Hence approximate $f(1998)$:

i	0	1	2	3	4	5
x_i	1980	1985	1990	1995	2000	2005
$y_i = f(x_i)$	440	510	525	571	500	600

(b). (i). Write down the general form of the Trapezoidal rule.

(ii). The acceleration of a rocket launched at time t = 0 from rest from the ground is measured and tabulated below.

Time (s)	0	20	40	60
Acceleration (ms^{-2})	57.5	59.0	63.8	67.2

Use Trapezoidal rule to find the speed of the rocked after 60s.

(6). (a). Write down Simpson's rule to evaluate approximate value for the integral $\int_a^b f(x)dx$.

(b). The values of the function $f(x)$ are tabulated as follows.

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	0.0000	0.1679	0.2955	0.4118	0.5460

Use Simpson's rule to evaluate the following integrals,

$$(i) \qquad \int_0^{0.8} (1+x)^2 f(x)\, dx$$

$$(ii) \qquad \int_0^{0.8} [f(x)]^2\, dx$$

correct to four decimal places.

Course: MPZ 3231-Engineering Mathematics IA **Assignment No.03**
Academic Year – 2013/2014

(1). Consider the following functions

(a). $y = \frac{x^2-x}{x+1}$; $x \neq -1$

(b). $y^2 = \frac{x}{x-2}$; $x \neq 2$

(c). $y = \frac{x^2-x-12}{x+2}$; $x \neq -2$

(i). Write down the horizontal, vertical and slant asymptotes of the graphs of $y = f(x)$.

(ii). Find the sign of $\frac{dy}{dx}$ as x varies and draw the graphs of $y = f(x)$.

(2). Evaluate the following limits

(a). $\lim_{x\to 4} \frac{x^2-16}{x^2+x-20}$

(b). $\lim_{x\to 1} \frac{x^5-2x^3-4x^2+9x-4}{x^4-2x^3+2x-1}$

(c). $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

(d). $\lim_{x\to 0} \frac{e^{ax}-e^{-ax}}{\log_e[1+bx]}$

(e). $\lim_{x\to 0} \frac{e^x-e^{-x}-2x}{x-\sin x}$

(3). (a). If $z = f(x+ay) + F(x-ay)$, find $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y^2}$ and

hence prove that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$

(b). If $z = e^{k(r-x)}$, where k is a constant and $r^2 = x^2 + y^2$ prove.

(i). $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zk\ \frac{\partial z}{\partial x} = 0$

(ii). $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + 2k\ \frac{\partial z}{\partial x} = \frac{kz}{r}$

(c). If $z = f(x-2y) + F(3x+y)$, where f and F are arbitrary functions, and if

$$\frac{\partial^2 z}{\partial x^2} + a\frac{\partial^2 z}{\partial x \partial y} + b\frac{\partial^2 z}{\partial y^2} = 0$$

Find the values of a and b .

(4). (a). If $z = f(x,y)$ where $x = e^u cosv$, $y = e^u sinv$, show that

$$\left[\frac{\partial z}{\partial u}\right]^2 + \left[\frac{\partial z}{\partial v}\right]^2 = e^{2u}\left[\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial z}{\partial y}\right]^2\right]$$

(b). If $f(x,y) = \sqrt{y^2-x^2}\ \ \sin^{-1}\frac{x}{y} + \frac{x^2-y^2}{\sqrt{x^2+y^2}}$, show that

$$x\left[\frac{\partial f}{\partial x}\right] + y\left[\frac{\partial f}{\partial y}\right] + z\left[\frac{\partial f}{\partial z}\right] = f(x,y)$$

(5). (a). Find the equations of the straight line perpendicular to both the lines

$$\frac{x-1}{1}=\frac{y+1}{2}=\frac{z}{3}$$

and $x+2y-3z=0=2x-y-z-6$
and passing through their point of intersection.

(b). (i). Find the equation of the plane through the line

$$\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+1}{2}$$

and parallel to

$$\frac{x-2}{2}=\frac{y-1}{-3}=\frac{z+4}{5}$$

(ii). Show that the point $(2,1,-4)$ lies on the above plane.

(6). (a). Find the equations of the straight line perpendicular to both the lines

$$\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{3}$$

and

$$\frac{x+2}{2}=\frac{y-5}{-1}=\frac{z+3}{2}$$

and passing through their point of intersection.

(b). Prove that the straight line

$$\frac{x-4}{3}=\frac{y-1}{2}=z-3$$

intersects the line of intersection of the planes

$$x+y+2z=4 \; and \; 3x-2y-z=3$$

and find the equation of the plane which contains the two lines

Course: MPZ 3231-Engineering Mathematics IA **Assignment No.04**
Academic Year – 2013/2014

(1). (a). Let z_1 , z_2 be two non-zero complex numbers. Prove that

$$arg\left(\frac{z_1}{z_2}\right) = arg(z_1) - arg(z_2)$$

Let $z_1 = -1 + i$ and $z_1 = 1 + \sqrt{3}\, i$. Find $Re\left(\frac{z_1}{z_2}\right)$ and $Im\ \left(\frac{z_1}{z_2}\right)$.

Express z_1 and z_2 in the form $r\ (\cos\theta) + \ i\ \sin\theta$,

where $r > 0$ and $0 < \ \theta\ < \ \pi$.

Deduce that $cos\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$.

(b). Find all the complex numbers z such that $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ and

$arg(z-1)\ (z+1) = \ \pi.$

(2). (a). Let z_1 and z_2 be complex numbers. Show that in the usual notation that

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2Re\ (z_1\ \overline{z_2})$$

Let P_1, P_2 and P_3 be the points in the Argand diagram representing the complex numbers z_1, z_2 and z_3 respectively.

If $z_1 + \ z_2 + \ z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, deduce that the triangle $P_1P_2P_3$ is equilateral.

(b). Prove that, if $Re\left(\frac{z-2i}{z+4}\right) = 0$ then the locus of z is a circle with radius $\sqrt{5}$.

Find its centre

If $Im\left(\frac{z-2i}{z+4}\right) = 0$ find the least value of $|z-1|$.

(3). (a). Prove that

(i). $|Re\, z| \leq |z|\ for\ all\ z \in \mathbb{C}$ and

(ii). $|z_1 + z_2|^2 = |z_1|^2 + 2\, Re(z_1\, \overline{z}_2) + |z_2|^2\ for\ all\ z_1, z_2 \in \mathbb{C}$.

Hence, prove that

(iii). $|z_1 + z_2| \leq |z_1| + |z_2|$ and

(iv). $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|\ for\ all\ z_1, z_2 \in \mathbb{C}$.

(b). Show that if $|z| = 1$ then $\left|\frac{z+1}{z^2+z-6}\right| \leq \frac{1}{2}$.

(4). (a). State De Moivre's Theorem for a positive integral index.

(i). Using De Moivre's Theorem, show that

$$\frac{\cos 4\theta}{\cos^4 \theta} = 1 - 6\, tan^2\, \theta + tan^4\theta$$

for $\theta \in \mathbb{R} - \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$.

Hence obtain the value of $tan\left(\frac{\pi}{8}\right)$

(ii). Express $\frac{1}{2}\left(\sqrt{3} + i\right)$ in the form cosθ + i sinθ, where $\theta \in \mathbb{R}$ and hence find the value of $\left(\frac{\sqrt{3}+i}{2}\right)^{2011}$.

(b). Find the fifth roots of -1 and hence, by considering the solution of the equation

$z^5 + 1 = 0$, prove that $cos\frac{3\pi}{5} + cos\frac{\pi}{5} = \frac{1}{2}$.

(5). (a). There are two modes of transporting material from Colombo to a city in the south, namely, by land or sea. Land transportation may be by rail or highway. About 70% of the material is transported by land and the rest is transported by sea. Also 60% of all land transportation is by highway and the rest by rail. The percentages of damaged cargo are 10 % by highway, 6 % by rail and 5% by sea.

(i) What percentage of the total cargo may be expected to be damaged?

(ii) If a damaged cargo is received what is the probability that it was shipped by

(A) Land?

(B) Sea?

(C) Rail?

(b). Suppose that Y possesses the density function

$$f(y) = \begin{cases} cy \,; 0 \leq y \leq 2 \\ 0 \,; \quad o/w \end{cases}$$

(i) Find the value of c that makes $f(y)$ a probability density function.
(ii) Find the cumulative distribution function $F(y)$.
(iii) Graph of $f(y)$ and $F(y)$.
(iv) Find $E(y)$ and $V(y)$.

(6). (a). In testing the lethal concentration of a chemical found in polluted water it is found that a certain concentration will kill 20% of the fish that are subjected to it for 24 hours. If 20 fish are placed in a tank containing this concentration of chemical, find the probability that after 24 hours,

(i) exactly 14 survive
(ii) at least 10 survive
(iii) at most 16 survive

(b). In a particular department store customers arrive at a checkout counter according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities?

(i) that no more than three customers arrive?
(ii) that at least two customers arrive?
(iii) that exactly five customers arrive?